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Analyzing Epistemological Learning Obstacles on Limit Concept Using AVAE Framework in Prospective Mathematics Teacher

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ABSTRACT

This study aims to identify the epistemological learning obstacles experienced by prospective mathematics teacher students in understanding the concept of limits in Calculus 1 lectures and the concept of limits. The primary focus of the analysis was conducted based on the categories of AVAE ERRORS (ARITH, VAR, AE, and EQS) related to the student's algebraic ability. The method employed was qualitative research involving 38 prospective mathematics teachers from one of the universities in West Java. Data was collected through written tests and confirmation interviews. The results showed that the most dominant epistemological learning obstacles were in the categories of AE (errors in algebraic expressions) and ARITH (arithmetic errors). These findings suggest that students struggle to apply prerequisite knowledge effectively to more complex limit problems. Therefore, a learning approach is needed that emphasizes not only algebraic procedures but also a deep conceptual understanding of the concept of limits.

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INTRODUCTION

Calculus is one of the foundations of mathematics and its application (Moru, 2009; Puspita et al., 2023a; Sulastri et al., 2022). Calculus is also indispensable in various scientific and technological developments (Beyene, 2023; Biza et al., 2022; Lyle et al., 2022). In the Mathematics Education Study Program at one of the universities in West Java, students have been required to take Calculus lectures since the first semester, starting with the Calculus 1 course. The basic concept of Calculus has been included in the high school curriculum, especially for students in the science program, as outlined in the 2013 Curriculum, or for students who choose Advanced Mathematics subjects in the Independent Curriculum. However, the findings in the field identified that students are not optimal in taking Calculus courses (Ismail & Bempah, 2018; Lyle et al., 2022; Musyrifah et al., 2022; Puspita et al., 2023b; Sulastri et al., 2022).

Learning obstacles are disturbances experienced by students, both from internal and external factors that cause them difficulties in following the learning process (Brown, 2008). Brousseau (2002) adapted the term "obstacles" from the theory put forward by Gaston Bachelard and Jean Piaget, related to the concept of "errors." Brousseau (2002) argues that obstacles play a complex, uncertain, and often unpredictable role yet remain an essential element in the process of knowledge formation. These obstacles are considered integral to the learning experience in understanding a concept (Booth, 2014; Rahmi & Yulianti, 2022; Sadiah et al., 2024) . In other words, obstacles in learning are inevitable because they are an essential aspect of the learning process. The difficulties students experience when working on mathematical problems are a sign of a learning obstacle (Booth, 2014; Brousseau, 2002), so identifying learning obstacles can be initiated by analyzing difficulties and errors. (Brousseau, 2002) distinguishes learning obstacles into three types: ontogenic obstacles (related to mental readiness to learn), epistemological obstacles (related to students' knowledge that has a limited context of application), and didactical obstacles (related to learning or teaching resources) that can occur during the learning process. Students' mistakes in answering the questions given can indicate the existence of epistemological learning obstacles (Brousseau, 2002).

Understanding these learning obstacles is crucial because weak mastery of the prerequisite material can lead to difficulties in building a deeper understanding of concepts (Booth, 2014). The lack of knowledge of these basic concepts not only hinders the lecture process but also affects students' ability to master the material at a higher level (Fuadiah et al., 2016). Without a solid foundation, students often struggle to connect new concepts with previously studied material, resulting in a less-than-optimal learning process. This emphasizes the importance of lecturers or educators in designing learning that is appropriate and tailored to the level of ability of their students based on the findings of learning obstacles (Nurhayati et al., 2023; Utami et al., 2022).

In calculus lectures, students require strong algebraic skills to optimize their understanding (Köğce, 2022). Therefore, when analyzing students' learning obstacles in calculus lectures, it is necessary first to assess the algebra skills that students possess. This can be identified by analyzing student errors on algebra-related problems. One of the categories of mistakes that can be used in issues related to algebra is the AVAEM category, consisting of ARITH, VAR, AE, EQS, and MATH (Putri et al., 2024; Ulfa et al., 2024).

The AVAEM category is an error category compiled explicitly in the field of algebra (Jupri et al., 2014). ARITH (arithmetic) is a mistake in performing arithmetic operations related to operations, rules, and properties; VAR (variable) is a student's mistake related to understanding variables; AE (algebraic expression) is a student's mistake in understanding algebraic expressions, EQS (equal sign) is a student's mistake in understanding the difference in meaning of the "=" sign, and the last is MATH (mathematization) is a student's mistake in doing mathematics (Jupri et al., 2014).

Previous research analyzing learning obstacles in calculus lectures has been conducted extensively, including a study by Puspita et al. (2022) that identified several types of learning obstacles related to chain rules, a topic in calculus. In addition, Musyrifah et al. (2022) conducted research on the analysis of learning obstacles for prospective mathematics teacher students in the context of derivative concepts. (Susilowati, 2021) analyzed student errors in solving Advanced Calculus problems. (Mutahharah et al., 2022) diagnosed learning difficulties experienced by high school students in learning the limits of algebraic functions. Meanwhile, Moru (2009), Sulastri et al. (2022), and Beyene (2023) specifically researched and analyzed learning obstacles to the concept of limits.

The concept of limit is one of the first materials given in the Calculus 1 lecture and becomes the basis for learning the following material. Based on the reference of previous studies, there has been no research related to learning obstacles to limit material, focusing on epistemological learning obstacles and grouping them based on the AVAEM category. Therefore, this study will focus on analyzing the epistemological learning obstacles encountered when studying the concept of limits and classifying them based on the AVAEM category. However, the categories of errors used by the researcher in this study are only four categories of errors, namely ARITH, VAR, AE, and EQS (AVAE), because the topic in calculus lectures discusses more symbolism in mathematics, so the test questions given do not require students to do the mathematical modeling stages.

While numerous studies have explored learning obstacles related to the concept of limits, none have systematically utilized the AVAE classification to organize their findings. This study places particular emphasis on analyzing epistemological learning obstacles and classifies them using the AVAE framework. The application of the AVAE categories is expected to provide a more structured basis for determining appropriate instructional interventions and reinforcement strategies to address the identified learning obstacles effectively.

METHOD

This research is qualitative. The participants in this study are 38 prospective mathematics teacher students who have obtained the Calculus 1 course. This research focuses on analyzing learning obstacles that are epistemological in nature. The purpose of this study is to identify the epistemological learning obstacles experienced by prospective mathematics teacher students in learning the concept of limits and classify them based on the AVAE category—data in the form of student answers to limit material questions collected through written tests.

The analysis procedure consisted of (1) recording all student answers, (2) identifying patterns of errors that appeared, (3) sorting out errors indicated as epistemological learning obstacles, and (4) confirmatory interviews with students to confirm the indicated error patterns. The head of the Mathematics Education study program validated the test instrument in this study, while the research data were validated by a senior lecturer responsible for teaching the calculus course.

RESULTS AND DISCUSSION

In this study, questions were given in the Calculus I lecture limit material. Students are asked to determine the limit value given. The questions are listed in Table 1 below.

Table 1. The questions

Question number	Question
(a)	Limits in infinity $\sqrt{2x^2 - x + 4} - \sqrt{2x^2 - 5}$)
(b)	Limits of trigonometric functions $\frac{(2x)-1}{4x^2}$
(c)	Limit of indeterminate form $\frac{x-5}{\sqrt{x^2-25}}$

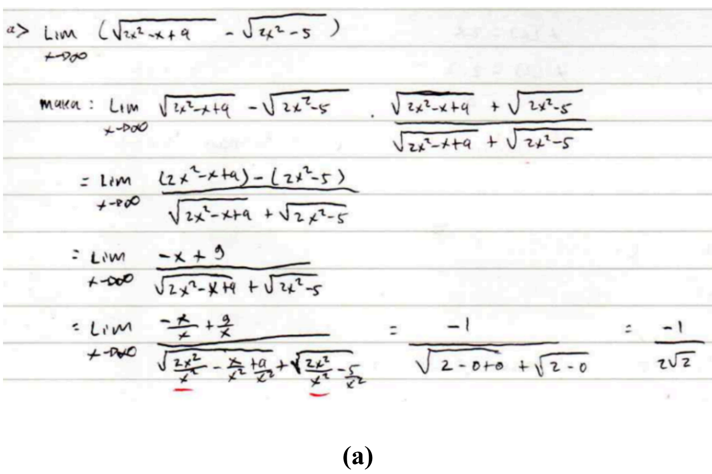
From the test given, as many as 17 students answered question number (a) correctly, 14 students answered part correctly, and seven students answered incorrectly. For question number (b), as many as 15 students answered correctly, 17 students answered some correctly, and six students answered incorrectly. Question number (c) was answered correctly by 19 students, partially correctly by 11 students, and responded incorrectly by eight students. Table 2 shows the percentage of students who answered correctly, partially true, and incorrectly for each question number.

Tabel 2. Percentage of students answer correctly, partially correct, and incorrectly

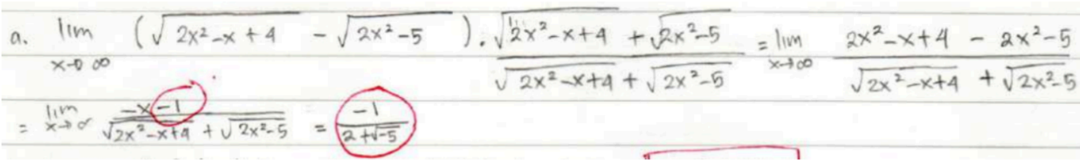
Question number	Correct (%)	Partially correct (%)	Incorrect (%)
(a)	45	37	18
(b)	39	45	16
(c)	50	29	21

Findings in Responds to Question (a)

Figure 1 shows students' responds to the question of the concept of limits in infinity. Several forms of error were identified in the responds to this question.



(a)



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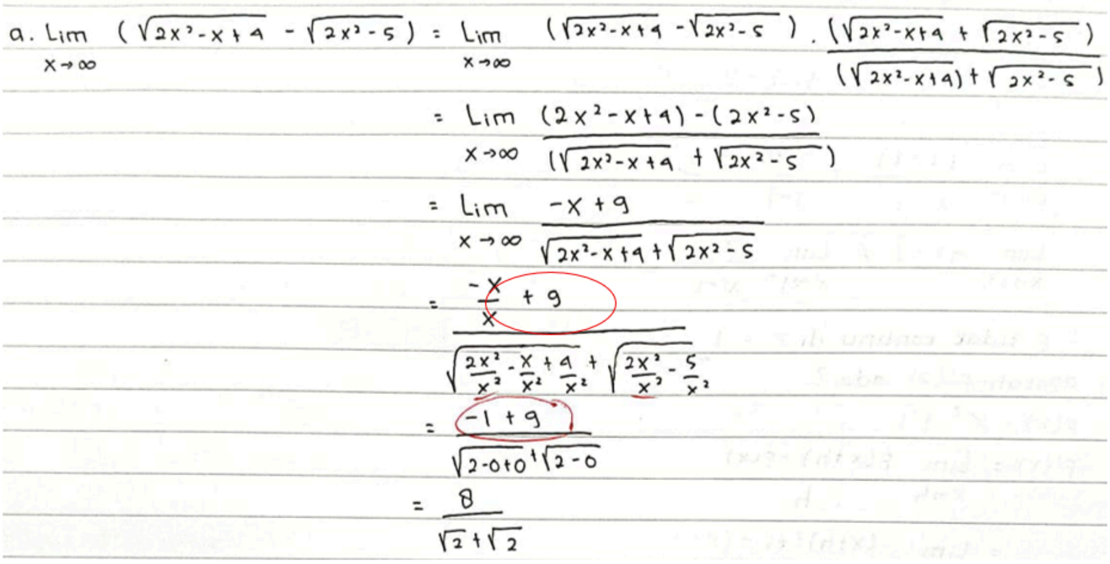

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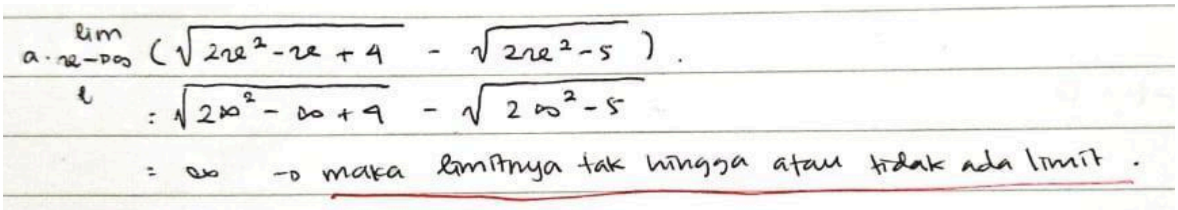


(d)

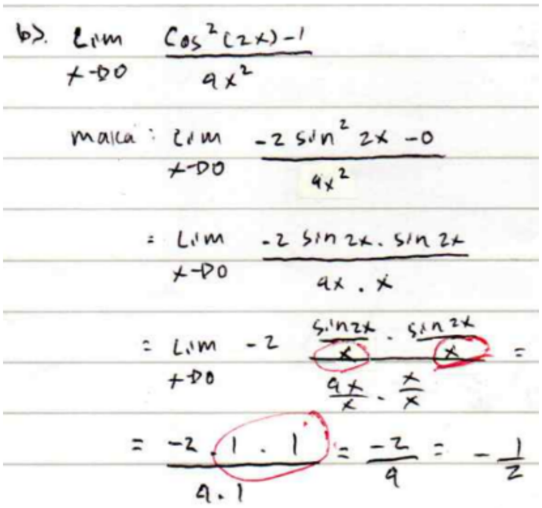
Figure 1. Student error findings on the limits of function in infinity

In Figure 1(a), students take the completion step inappropriately. This step will find an obstacle if the question is changed from x to negative to infinity ($x \to -\infty$) because the answer will be wrong if you continue to use the steps as the student did. Generally, the steps to work on problems like this are acquired when students are still in high school. In Figure 1(b), students make a mistake in performing algebraic calculation operations when simplifying the form of expression that appears, likewise, in Figure 1(c). The student made a mistake when dividing each quarter by x, which should have resulted in $-1 + \frac{9}{x}$. Instead of $-1 + 9$. This results in an error in the next step. These three errors are related to performing algebraic form calculation operations and can be categorized as arithmetic errors. Although this type of error has not been identified in previous studies concerning the concept of limits, it has frequently been observed in algebra-related topics, as documented in the works of Jupri et al. (2014) and Ulfa et al. (2024).

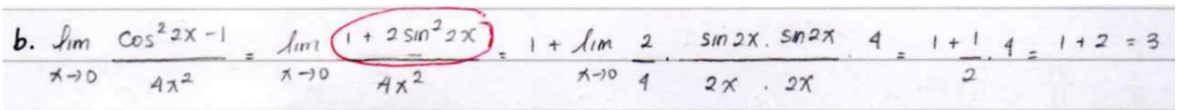
Meanwhile, in Figure 1(d), the student immediately substitutes $x = \infty$, which results in a limit value as ∞ and states that there is no limit value for the question involving the number. This student did not realize that the form of the problem could not be solved by the substitution method. This finding was also identified in a study conducted by Beyene (2023).These errors can be categorized both as ARITH or AE.

Findings in Responds to Question (b)

The error findings identified from the responds to question (b) are shown in Figure 2. The following are the answers given by the students.



(a)



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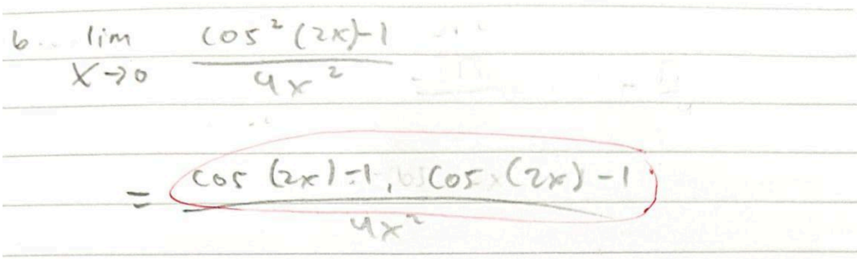
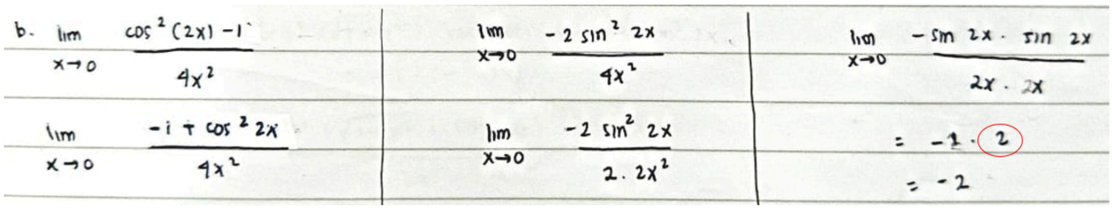


(c)



(d)

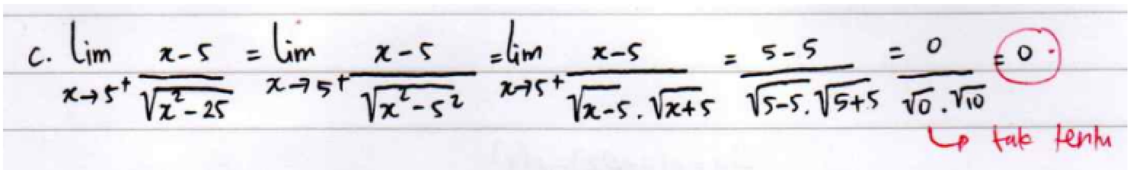
Figure 2. Finding student errors in the limits of trigonometry functions

In Figure 2(a), students write down the exact answer steps, just like in the example given earlier. Solving this problem can be done in shorter steps. Based on the interview conducted with the student, the student stated that this method is the way taught in class, and the person concerned does not know an easier way to solve it. These findings show that students' analysis of the questions given is still limited to the sample answers that have been discussed. Figure 2(b) and (c) show the students' mistakes in changing the trigonometric form to another equivalent form. It leads to errors in completing the next step, resulting in an incorrect final answer. Meanwhile, in Figure 2(d), students mistakenly apply the nature of trigonometric limits and state the result of the limit form $\frac{sinsin\, 2x}{2x}$ as 2. Therefore, the final answer given is also incorrect. Obstacles in solving problems involving trigonometric functions were likewise observed in the study conducted by Köğce (2022), however, the test on that research was on integration problems rather than on limits.

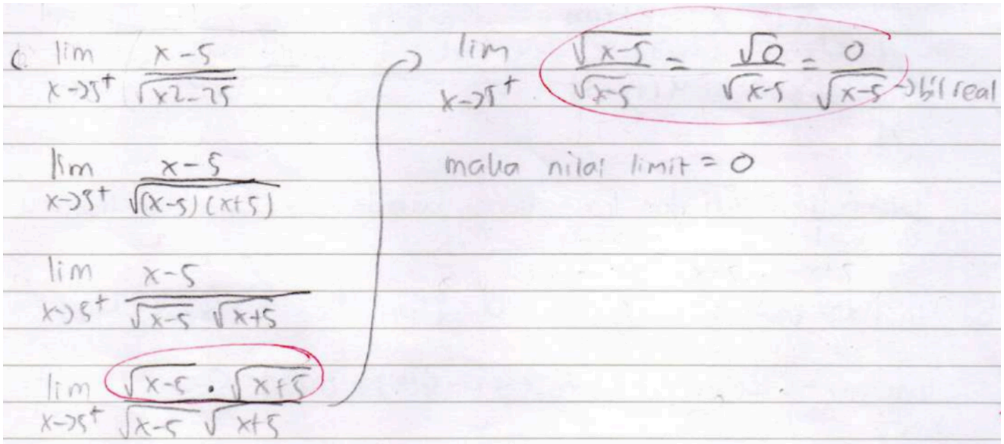
The findings of these errors can also be categorized as ARITH. Students often have a limited understanding of performing simpler algebraic operations when solving limit forms and frequently make errors when simplifying limit forms.

Findings in Responds to Question (c)

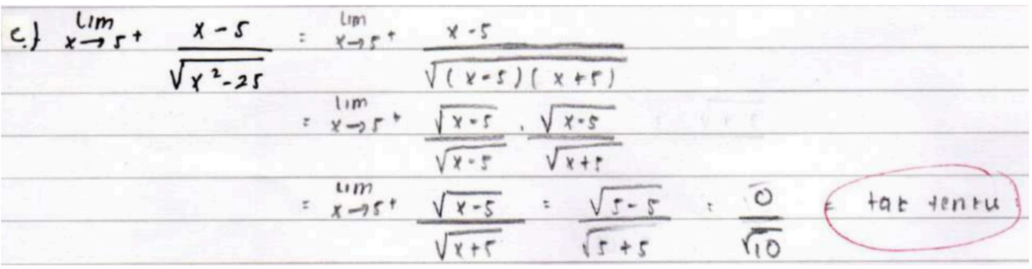
The answers shown in Figure 3 are the finding of student errors in solving question (c). Here are some conclusions of the students' answers.



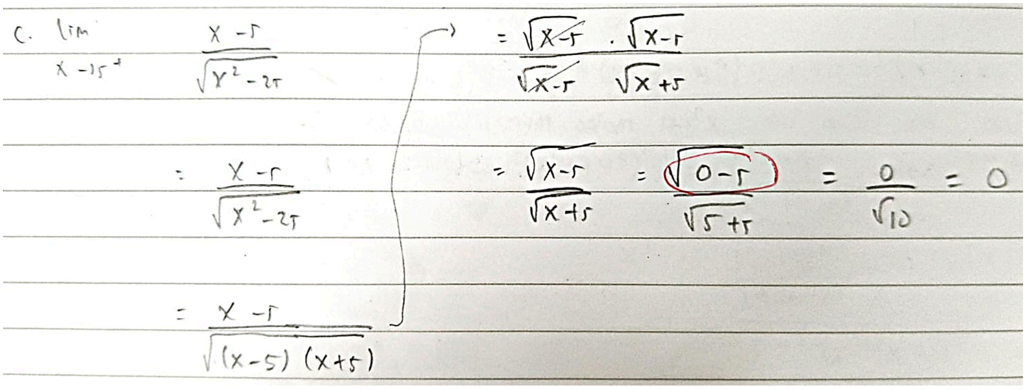
(a)



(b)



(c)



(d)

Figure 3. Finding student mistakes in solving indeterminate form limits

In the student's answer shown in Figure 3(a), in addition to the final result, the student also made an error in selecting the work step. In the process of completing the function limit, direct substitution is not allowed if it results in an indeterminate final result, one of which is $\frac{0}{0}$. The method used should be a factorization method. Students first factor the numerators and denominators, then simplify the resulting algebraic forms before making substitutions. However, this student still made substitutions before simplifying the form of the question and writing the final result as 0. Error in stating $\frac{0}{0}$ as zero can be categorized as ARITH, while errors in choosing a method to solve the limit can be categorized as AE.

In Figure 3(b), students are wrong in factoring the numerator. This error in factorization was also found in a study conducted by Ismail & Bempah (2018) and Mutahharah et al. (2022). In addition, in the final result, the student stated that the limit value was zero and did not realize that if the value was $x \to 5^+$ substituted into the denominator will result in the form $\frac{0}{0}$. This error can also be categorized as ARITH, where students make mistakes in factoring and deducing the final result.

In contrast to the answer shown in Figure 3(c). The student is correct in factoring and simplifying the form of the limit given. The final result written is also correct, namely,0-,10... However, the student was wrong when he concluded that $\frac{0}{\sqrt{10}}$ is an indeterminate form. This error can be categorized as ARITH as well as AE. Students cannot simplify operations that appear in the form $\frac{0}{\sqrt{10}}$. They can be categorized as ARITH. Meanwhile, students' mistakes in recognizing indeterminate forms can be categorized as AE. This error has not been reported in prior studies that are relevant to this research.

The student with the answer in Figure 3(d) made a mistake when making a substitution, where the student substituted x on the numerator with zero, where it should have been $x \to 5^+$. However, the student still gave the right answer. Based on the interview, the student had calculated that the limit value was 0 but was not careful when writing the answer. This error can be categorized as ARITH.

Error Finding Categories

After the findings of the errors have been analyzed, the categories for each form of error can be seen in Table 3.

Table 3. Error categories

Error Findings	Category
Mistake in simplifying the form of the question	ARITH
Errors in performing algebraic calculation operations	ARITH
Mistake in substitution $x \to \infty$	ARITH, AE
Limitations in analyzing simpler problem-solving steps	ARITH
Mistakes in utilizing the trigonometric limit properties	ARITH

Mistakes in converting trigonometric forms into other equivalent forms	ARITH
Confused in concluding $\frac{0}{0}$	ARITH
Mistakes in recognizing indeterminate form	ARITH, AE
Wrong in choosing the limit settlement method	ARITH

CONCLUSION

This study identifies various mistakes of prospective mathematics teacher students in solving limit problems in the Calculus I course. These findings are in line with previous research that also showed the dominance of ARITH and AE errors, as well as procedural errors within limits that indicate epistemological learning obstacles. This shows the need for learning interventions that focus on understanding the concept of limits in-depth, not just algebraic procedures. Error analysis with the AVAE framework provides a structured picture that can be used to develop a more optimal learning design and be able to overcome learning obstacles that are predicted to arise. In further research, it is necessary to analyze other types of learning obstacles so that they are considered in developing a better learning design.

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