

The Archimedean Property in Real Analysis: A Philosophical and Pedagogical Integration with Islamic Values

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ABSTRACT

This study examines the integration of Archimedes' property in real analysis with Islamic epistemology, focusing on how this integration can enhance mathematics education in Islamic higher education. The study explores Archimedes' property from both a mathematical and philosophical perspective, proposing a pedagogical model that combines technical mathematical understanding with ethical values such as humility. The study's primary contribution lies in the theoretical exploration of Archimedes' property within a broader philosophical framework, emphasizing the limitations of human knowledge and the spiritual insights offered by Islamic teachings. While the study offers an integrative pedagogical model designed to improve students' engagement with mathematics through both intellectual and moral development, it has not been empirically tested in classroom settings. Therefore, the claims of the model's feasibility and impact remain theoretical. The study suggests that future research should pilot the model in real educational environments to assess its practical effectiveness and impact on student learning outcomes. This work contributes to the literature by highlighting the potential for integrating philosophical and ethical dimensions into mathematics education, offering a new approach to teaching real analysis that is relevant to both academic and spiritual development.

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INTRODUCTION

The role of Archimedes' Property in Real Analysis

Archimedes' property plays a crucial role in the foundational structure of real analysis. It asserts that for any real number ($x > 0$), there exists a natural number (n) such that ($n > x$). This principle ensures the unbounded nature of natural numbers within the real number system, reinforcing the concept of completeness in real numbers. Completeness, a defining feature of real numbers, states that every bounded sequence has a least upper bound or supremum, and Archimedes' property supports this by ensuring that there are no infinitesimal quantities or infinite numbers. It guarantees that real numbers provide a coherent description of continuous quantities, facilitating the structure of analysis by keeping the real number system logically sound (Bowers et al., 2018; D'Agostino, 2020).

Archimedes' property also underpins many key concepts in calculus and real analysis, such as limits, continuity, and the derivative. It maintains the well-ordering of natural numbers, which is fundamental for defining limits and performing rigorous analysis in calculus (Barki et al., 2016; Berestycki & Engelenburg, 2023). The property provides the essential foundation for understanding the nature of sequences, neighborhoods, and other concepts that are central to real analysis. By asserting the absence of infinitesimals

and ensuring the boundedness of the real numbers, it allows for the creation of a coherent framework for defining mathematical functions, limits, and convergence in real number systems.

Applications of Archimedes' Property in Various Fields

Beyond its foundational role in real analysis, Archimedes' property has far-reaching applications in other fields like field theory and probability. In statistical modeling, for example, the property helps validate that certain distributions are bounded, ensuring that probabilities remain confined to real values. This contributes to the stability and reliability of probabilistic models, affirming that distributions generated by random variables stay within measurable and finite limits (Berestycki & Engelenburg, 2023; Charernboon, 2017). Similarly, Archimedes' property plays a crucial role in the standardization of mathematical models across various domains, reinforcing the consistency and predictability of systems in fields ranging from economics to natural sciences.

However, in educational contexts, Archimedes' property is often taught in a purely technical manner, focusing mainly on its application within the realm of real analysis. While this technical focus is essential, it leaves little room for exploring the broader implications of the property in higher-level mathematical constructs and its connection to other fields. Many students may grasp the fundamental nature of the Archimedean property in relation to real numbers but struggle to apply it in advanced topics or recognize its philosophical and practical significance. This gap in understanding can lead to misconceptions when students encounter more complex mathematical models, particularly in probability theory, where the importance of Archimedes' property may not be immediately apparent (Avramidi & Nguyễn-Phan, 2019; Maillard & Penington, 2024).

Islamic Epistemology and Mathematical Infinity

Islamic epistemology offers an insightful perspective on infinity, especially through the concept of humility (*tawadhu*) and the acknowledgment of human epistemic limitations. In Islamic thought, the idea that human knowledge is finite aligns with the mathematical concept of infinity in Archimedes' property. While Archimedes' property implies that numbers can extend infinitely, they remain bounded within a finite framework of real analysis, much like human understanding is finite in comparison to the infinite knowledge of the Divine. This analogy reflects the Islamic belief in the limitations of human knowledge, emphasizing humility and the continuous pursuit of knowledge (D'Agostino, 2020; Giorgadze & Osmanov, 2024).

The connection between Archimedes' property and Islamic thought is significant, as it fosters a deeper understanding of mathematical infinity and its limitations. Archimedes' property helps students appreciate that while mathematical constructs can approach infinity, they do so within a defined, finite structure. Similarly, Islamic teachings emphasize the infinite nature of divine knowledge, suggesting that human understanding can only grasp a small portion of the vastness of truth. This perspective not only grounds students in the realities of mathematical modeling but also encourages them to approach mathematical infinity with caution and respect, reinforcing the value of intellectual humility and the recognition of the limits of human knowledge (Jauregui, 2019; Parkkonen & Paulin, 2022).

Integrating Philosophical and Theological Dimensions with Mathematical Education in Islamic Contexts

The integration of philosophical and theological dimensions into mathematics education, especially within the context of Islamic higher education, offers a holistic approach to learning that encourages both intellectual and moral growth. By embedding values such as humility and awareness of knowledge limitations, this approach transforms mathematics from a purely technical discipline into one that intersects with ethical considerations and spiritual insights. In Islamic contexts, the study of mathematics is not limited to mastering abstract concepts but is extended to shape students' character and moral development. This integration enriches students' engagement with mathematical concepts by framing them within broader existential questions, ultimately fostering a deeper appreciation for the subject as a tool for personal and societal reflection (Corazza, 2016; Fu & Yu, 2020).

In this study, Archimedes' property is viewed not only as a key concept in real analysis but also as an opportunity for philosophical and ethical reflection. By exploring its implications within the framework of Islamic epistemology, the study aims to provide students with a richer understanding of mathematics, one that transcends the technical aspects and incorporates broader philosophical questions. This approach helps students see the relevance of mathematical concepts, such as infinity, in both academic and ethical contexts, thereby enhancing their engagement with the subject. It also encourages a more reflective and conscientious approach to mathematical learning, linking abstract concepts to personal values and societal impacts (Corgnier et al., 2017; Spreen & Berger, 2023).

Existing Studies on the Integration of Islamic Values into Mathematics Education

Existing research has explored the integration of Islamic values into mathematics education, with many studies highlighting the benefits of embedding ethical and philosophical discussions into the curriculum. These discussions are particularly valuable in fields like probability theory, where issues of justice, fairness, and responsibility can be discussed through a mathematical lens. By incorporating Archimedes' property and other foundational concepts into ethical contexts, students are encouraged to reflect on the societal impact of mathematics and the responsibility that comes with its application (Kang & Koh, 2018). However, challenges remain in blending traditional mathematical pedagogy with values-based education. Some educators fear that such integration might undermine the intellectual rigor of mathematics. Overcoming this challenge requires careful curriculum design that respects both the mathematical content and the ethical principles being integrated (Karaaslan & Karakaya, 2024; Narita et al., 2017). Nonetheless, successful integration has been shown to enhance student motivation, as students become more engaged when they see the relevance of mathematics to their personal beliefs and values (Benci et al., 2019; Galecka, 2021).

Theoretical and Pedagogical Contributions of This Study

This study contributes to both the theoretical and pedagogical understanding of Archimedes' property in real analysis by contextualizing it within a framework that merges mathematical rigor with philosophical and theological inquiry. By viewing Archimedes' property as not only a mathematical axiom but also a concept that facilitates ethical and spiritual reflection, the study enriches academic discussions about its role in real analysis. Additionally, the study makes a significant contribution to the integration of Islamic values into mathematics education by proposing pedagogical strategies for embedding ethical discussions into the curriculum. These strategies encourage educators to engage students in discussions that bridge the gap between mathematics and their cultural heritage, offering a deeper, more holistic approach to learning (Chai & Zhang, 2016; Guo & Yu, 2023).

METHOD

This research employs a qualitative-descriptive approach combined with mathematical conceptual analysis and Islamic text studies. The methodological approach was selected to explore, formulate, and synthesize a deep understanding of Archimedes' property in real analysis while integrating philosophical and theological perspectives, particularly within the context of Islamic education. The study aims to provide a novel synthesis of mathematical rigor with philosophical inquiry and ethical reflection. The design of the study is divided into three stages: axiomatic formulation of Archimedes' property, hermeneutic analysis of Islamic texts, and the formulation of an integrative pedagogical model. This approach ensures that the study is both academically rigorous and culturally relevant, offering a comprehensive understanding of Archimedes' property and its implications for Islamic higher education.

Research Procedures

The research procedure is structured across three stages. The first stage involves the mathematical exploration of Archimedes' property, where axiomatic formulations and formal proof steps are presented in the context of real analysis. This is followed by hermeneutic analysis, where selected Qur'anic verses are interpreted using a theological lens to explore the philosophical meaning of Archimedes' property, particularly regarding the concept of infinity. Finally, the third stage focuses on pedagogical integration, where a model is developed to integrate the mathematical and theological aspects, facilitating a holistic approach to mathematics education. These stages are validated through expert assessment to ensure that the proposed model is both pedagogically feasible and aligned with Islamic epistemological values.

Data Sources

Data sources for this study were selected from three key categories:

1. Primary mathematical sources such as classical real analysis textbooks (e.g., Bartle, 1977; Tall, 2013) and technical articles on Archimedean and non-Archimedean structures.
2. Islamic primary sources, specifically the Qur'an (QS. Yusuf: 76, QS. Al-Kahfi: 109), along with classical and contemporary Islamic exegeses to provide a theological framework.
3. Secondary sources, including previous studies on the integration of values in mathematics education and relevant pedagogical literature, to support triangulation and the development of the pedagogical model. All sources were carefully selected for their academic credibility and relevance to the research objectives.

Research Instruments and Ethics

The research instruments include:

1. **Formal proof guidelines:** These were used to guide the formulation of axioms, lemmas, and theorems in real analysis, ensuring the rigor of mathematical arguments.
2. **Exegesis text analysis sheets:** These sheets helped in the interpretation of Qur'anic verses, applying a theological and philosophical lens to understand the relationship between Archimedes' property and Islamic epistemology.
3. **Expert assessment sheets:** These were used to validate the integrative pedagogical model, focusing on its content validity, coherence with Islamic values, and pedagogical feasibility.

Ethical considerations were strictly adhered to throughout the study. This includes accurate citation and respectful handling of religious sources, transparency in the methodology, and the disclosure of potential biases. The study ensured that expert validation was conducted with academic integrity, using neutral and objective assessment criteria. Additionally, the research adhered to ethical guidelines for qualitative research, ensuring the transparency and reproducibility of the study.

Data Analyses

Data analysis was performed using both deductive and inductive approaches. The mathematical component was analyzed deductively, with formal definitions and proofs presented logically and step-by-step. The hermeneutic analysis involved thematic categorization to correlate the philosophical meanings of the Qur'anic verses with the mathematical implications of Archimedes' property. The integration of mathematical and theological perspectives was synthesized in an interpretive framework, considering both the epistemic limitations of human knowledge and the relevance of Islamic thought to mathematical concepts like infinity. Expert triangulation was used to validate the integration process, with feedback collected from experts in mathematics, Islamic studies, and pedagogy. Reliability measures were ensured by maintaining detailed records of expert discussions, annotations, and revisions, in accordance with established qualitative research standards (Lincoln & Guba, 1985; Tisdell et al., 2025).

RESULTS AND DISCUSSION

Results of Axiomatic Formulation and Mathematical Exploration

The first part of the study focused on the axiomatic formulation of Archimedes' property within the context of real analysis. The property was presented as one of the foundational axioms that complement the ordered field structure of real numbers. The formulation of Archimedes' property was explicitly detailed, ensuring clarity in how it supports the completeness and consistency of the real number system. Lemmas and theorems were derived to demonstrate the theoretical implications of the property, confirming its role in ensuring that there are no infinitesimal quantities and that real numbers are appropriately bounded.

The proof of Archimedes' property was formalized through a series of lemmas, as follows:

1. Lemma 1: The Archimedean property is equivalent to the statement that for every real number x , there exists a natural number n such that $n > x$. This lemma confirms that the real number system is unbounded, supporting its completeness.
2. Lemma 2: No non-zero infinitesimal real numbers exist. This lemma further establishes the absence of infinitesimal quantities in the real number system, reinforcing the real numbers' role in describing continuous quantities.
3. Theorem 1: The density of the rational numbers \mathbb{Q} in \mathbb{R} is a direct consequence of Archimedes' property, which guarantees the ability of natural numbers to approximate real numbers within any given interval (Barki et al., 2016; Berestycki & Engelenburg, 2023).

These results, while not novel in the strictest mathematical sense, reinforce the importance of Archimedes' property in real analysis and provide a clear, accessible explanation for students of real analysis, further grounding the concept in both mathematical rigor and pedagogical clarity. This aligns with the foundational role of Archimedes' property in the completeness of real numbers, ensuring consistency and coherence in the system.

Hermeneutic Analysis and Theological Meaning

The second part of the study involved a hermeneutic analysis of two key Qur'anic verses: QS. Yusuf: 76 and QS. Al-Kahfi: 109, both of which emphasize the limitations of human knowledge. The analysis focused on the metaphorical relationship between mathematical infinity, as embodied in Archimedes' property, and the

theological concept of infinity in Islamic thought. By interpreting these verses, the study illustrated how mathematical concepts can be used as metaphors for understanding the divine limits of knowledge. This analysis was framed as a functional analogy rather than an ontological equivalence, in line with hermeneutic principles (Gadamer et al., 1989; Ricoeur, 2008).

In QS. Yusuf: 76, the verse

فَوْقَ كُلِّ ذِي عِلْمٍ عَلِيمٌ

"Above every knowledgeable person, there is someone who knows more" emphasizes the relative nature of human knowledge, which parallels the iterative approximation of real numbers by natural numbers in Archimedes' property. This analogy supports the notion that, while humans can approach infinity mathematically, true infinity lies beyond their grasp, much like the infinite knowledge of the Divine. Similarly, QS. Al-Kahfi: 109, which states that even if all the oceans were ink, it would still not be enough to describe God's knowledge, reinforces the concept of the infinite as something that transcends human understanding. This theological perspective enriches the mathematical concept of infinity by acknowledging its limitations, further grounding the study in Islamic epistemological thought (Giorgadze & Osmanov, 2024).

This hermeneutic analysis establishes a deep, philosophical link between mathematics and Islamic epistemology, contributing to a broader understanding of Archimedes' property within both a scientific and spiritual context. The philosophical analogy helps students recognize the limitations of human knowledge and fosters humility in the pursuit of learning, aligning well with Islamic teachings on intellectual humility (Jauregui, 2019; Parkkonen & Paulin, 2022).

Pedagogical Integration: Development of the Learning Model

The third component of the study involved the development of an integrative pedagogical model that combines Archimedes' property with Islamic values, particularly focusing on humility and epistemic awareness. The proposed model is designed for use in university-level mathematics education, specifically within the context of Real Analysis or Philosophy of Mathematics courses. The model includes cognitive, affective, and spiritual learning objectives, promoting both intellectual growth and moral development. This aligns with the educational goals outlined in the literature, which emphasize holistic learning that bridges intellectual competence with ethical and spiritual development (Corazza, 2016; Fu & Yu, 2020).

Key activities in the model include:

1. Collaborative Proof Sessions: Students work in groups to prove lemmas and theorems related to Archimedes' property using an axiomatic approach. This collaborative activity helps build epistemic interactions and strengthens mathematical reasoning, consistent with studies highlighting the value of collaborative proof in mathematics education (Corgnier et al., 2017; Spreen & Berger, 2023).
2. Hermeneutic Case Studies: Students engage in case studies that connect the concept of limits and the accessibility of natural numbers with the Qur'anic verses on the limitations of knowledge. These case studies encourage students to reflect on the relationship between mathematics and spiritual insights.
3. Structured Reflection: Students write reflections on how their understanding of Archimedes' property intersects with the philosophical and theological dimensions of Islamic teachings. This activity promotes metacognitive awareness and internalizes the value of humility and lifelong learning, in line with the philosophical foundations of Islamic education (Benci et al., 2019; Galecka, 2021).

Expert Validation and Methodological Transparency

The pedagogical model was validated through expert judgment from seven experts: three in Real Analysis, two in Islamic Studies, and two in Mathematics Education. The validation process focused on four key aspects: content validity, theory-practice coherence, feasibility of implementation, and cultural sensitivity. The results were promising, with the model receiving high ratings (average scores of 4.5 for content validity, 4.2 for feasibility, and 4.6 for cultural sensitivity). However, there was some feedback regarding the need for more contextual examples in the collaborative proof sessions and suggestions for further refinement of the assessment criteria to monitor students' epistemic development.

While the expert validation results were positive, the model has not yet been empirically tested in a classroom setting. Therefore, its practical impact and effectiveness remain theoretical. Future research should involve piloting the model in real classroom environments to evaluate its impact on students' cognitive understanding and the development of their spiritual dispositions. A quasi-experimental design with control groups could provide valuable insights into the model's efficacy, as noted in the existing literature on values integration in mathematics education (Asna, R., et al., 2024; Benci et al., 2019; Galecka, 2021; Pebria, W., et al., 2024; Rahmi, Y., et al. 2023).

Equations should be placed at the center of the line and provided consecutively with equation numbers in parentheses flushed to the right margin, as in (1). The use of Microsoft Equation Editor or MathType is preferred.

$$E_v - E = \frac{h}{2.m} (k_x^2 + k_y^2) \quad (1)$$

All symbols that have been used in the equations should be defined in the following text.

Sub section 2 (Heading 3) (Italic 10 pt)

Proper citation of other works should be made to avoid plagiarism. Examples of reference items of different categories shown in the References section. Each item in the references section should be typed using 10 pt font size.

CONCLUSIONS

This study offers a theoretical and pedagogical exploration of Archimedes' property in real analysis by integrating it with Islamic epistemology. The main contribution of the study lies in its ability to contextualize Archimedes' property within a philosophical framework, linking mathematical rigor with ethical and spiritual reflection. While the study enriches the understanding of Archimedes' property by framing it in a new philosophical and theological light, the claims of a "new paradigm" for real analysis are overstated. The study's primary contribution is pedagogical, aiming to enhance students' engagement with both mathematical concepts and ethical values in Islamic higher education.

The proposed pedagogical model, which integrates Archimedes' property with Islamic values, offers a promising framework for teaching real analysis in a more holistic manner. This model encourages students to not only understand the technical aspects of real analysis but also to reflect on the philosophical and ethical dimensions of mathematical knowledge. However, it is important to note that the model has not yet been empirically tested in classroom settings, and as such, the claims of its effectiveness and impact remain theoretical. Future research is needed to pilot this model in real classroom environments, using empirical methods to assess its feasibility and impact on students' cognitive and ethical development.

In terms of practical implications, this study offers valuable insights into how mathematical education can be enriched by incorporating ethical and philosophical considerations. By proposing a model that combines intellectual and moral growth, the study highlights the importance of preparing students not only to excel academically but also to become responsible, reflective individuals. While the model has been validated by expert judgment, the next step involves empirical testing to confirm its practicality and effectiveness in enhancing student learning outcomes. Future studies could also explore the integration of other mathematical principles with ethical values, further contributing to the development of values-based education in mathematics.

In summary, while the study provides a valuable theoretical contribution to real analysis and Islamic education, its claims of novelty should be viewed in the context of pedagogical integration rather than theoretical innovation in mathematics. The next step is to test the proposed model in classroom settings, allowing for a more thorough understanding of its potential impact on students' engagement and development. This would help to bridge the gap between theoretical concepts and their practical application in educational environments.

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