

## Error Patterns in Simplifying Rational Algebraic Expressions among Rural Congolese Students

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### ABSTRACT

This study investigated the error patterns exhibited by rural Congolese students when simplifying rational algebraic expressions. An exploratory qualitative design within an interpretive research paradigm was employed. The participants were 28 Algebra I students enrolled in a rural secondary school in the Democratic Republic of the Congo during the 2025 academic year. Data were collected using a specialist-validated evaluative test comprising 10 simplification items. The 28 students completed all 10 items, producing 280 responses. Students' written solutions were examined item by item and classified using an adapted error framework encompassing symbolic, conceptual, reasoning, procedural, and technical difficulties. An error pattern was considered recurrent when it occurred in more than 30% of the participants' responses. Of the 280 responses, 136 responses (48.6%) contained at least one error, whereas 106 responses (37.9%) were correct and 38 responses (13.6%) were unanswered. The most dominant categories were technical errors and errors associated with insufficient prior knowledge of algebraic facts, skills, and concepts. Recurring difficulties included incorrect factorization, mismanagement of negative signs, inappropriate application of algebraic rules, deficient manipulation of symbols, and difficulties simplifying notable products. These findings indicate that students' errors were systematic and reflected both conceptual weaknesses and limited procedural fluency. Instruction should therefore emphasize explicit teaching of factorization, sign operations, algebraic equivalence, and step-by-step verification of symbolic procedures, particularly in rural and under-resourced learning environments.

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## INTRODUCTION

Algebra is a central component of school mathematics because it enables students to represent general relationships, reason with variables, and move beyond arithmetic procedures toward more abstract forms of mathematical thinking. Nevertheless, learning algebra remains challenging for many secondary school students, particularly when tasks require them to interpret symbolic structures and transform expressions while preserving their mathematical meaning. Simplifying rational algebraic expressions is especially demanding because it is not merely a procedural activity; students must coordinate factorization, distributive reasoning, sign and bracket operations, fraction properties, and the recognition of equivalent forms. Successful simplification therefore requires learners to regard expressions as structured mathematical objects rather than as strings of symbols to be manipulated mechanically. Previous studies have shown that students often experience difficulties interpreting variables, recognizing algebraic structure, and understanding the equal sign as a relational symbol rather than an instruction to calculate (Ardiansari et al., 2020; Qetrani et al., 2021). Weak understanding of equivalence is

particularly consequential because every simplification step must produce an expression equivalent to the original, subject to any relevant restrictions on the variables.

These conceptual difficulties are closely associated with recurring procedural errors in students' written algebraic work. Research has documented problems involving fraction equivalence, factorization and expansion, the order of operations, and the interpretation of negative signs and brackets (Barana, 2021; Eroğlu, 2023; Ottmar et al., 2023; Qetrani et al., 2021). Students may apply familiar rules without considering whether those rules are appropriate for the structure of the expression, resulting in invalid transformations, incorrect cancellation, sign changes, or non-equivalent simplified forms. Similar patterns have been identified in studies of algebraic manipulation, which report errors related to symbolic interpretation, procedural hierarchy, and the inappropriate application of rules and algorithms (Moru & Mathunya, 2022). Such errors are often systematic rather than accidental because they reflect deficiencies in prior knowledge, conceptual understanding, or procedural fluency. Consequently, examining students' solutions in detail can reveal not only whether an answer is incorrect but also the reasoning and misconceptions underlying the error. This error-based perspective provides an important foundation for understanding students' difficulties with rational algebraic expressions and for designing more responsive algebra instruction.

Prior mathematical knowledge plays a decisive role in students' success in simplifying rational algebraic expressions. Misconceptions about fractions, equivalent fractions, the fraction bar, and the coordination of numerators and denominators may persist into later algebra learning and interfere with operations on rational expressions (Chrysostomou & Christou, 2019). Students may also overgeneralize previously learned arithmetic or equation-solving procedures into contexts where those procedures are no longer valid. For example, learners may treat a simplification task as though it were an equation-solving task, combine unlike terms, or apply informal cancellation rules without attending to the multiplicative structure of the expression (Ardiansari et al., 2020; Barana, 2021). These patterns suggest that errors in rational-expression simplification are not simply computational slips but may reflect unstable conceptual schemas that have accumulated across earlier stages of mathematics learning. This makes error analysis particularly important because it can reveal how prior knowledge is being activated, misapplied, or incompletely reconstructed during algebraic activity.

Instructional practices may either reduce or reinforce these difficulties. Lessons dominated by teacher demonstration and rule-based procedures can limit students' opportunities to compare solution strategies, explain their reasoning, and justify why a transformation preserves equivalence (Chan et al., 2022). When learners are asked only to imitate procedures, misconceptions related to signs, brackets, fractions, and algebraic structure may remain hidden until they reappear in more complex tasks. By contrast, instruction that encourages discussion, multiple representations, and targeted scaffolding can help students articulate and revise their interpretations of symbolic expressions (Eroğlu, 2023; Mouhayar, 2021). These considerations are especially urgent in rural and under-supported educational settings, where learners may have fewer opportunities to access sustained conceptual support. Nevertheless, rurality itself should not be treated as a direct cause of error; rather, the relevant concern is how limited opportunities to learn, uneven prior preparation, and restricted instructional support may shape students' algebraic performance (Kinach, 2024; Mouhayar, 2021).

Despite the growing literature on algebraic misconceptions, structural thinking, and rational-expression learning, relatively little is known about the specific error patterns produced by students in rural Congolese secondary schools. Existing studies have largely examined learners in other national, linguistic, or institutional contexts, and few have combined item-level analysis with a structured classification of conceptual, procedural, symbolic, and technical errors in simplifying rational algebraic expressions. This contextual and analytical gap constitutes the novelty of the present study. Accordingly, this study aims to identify the error patterns exhibited by Algebra I students when simplifying rational algebraic expressions in a rural school in the Democratic Republic of the Congo and to classify these errors using an adapted framework based on Aguilera et al. (2020). By documenting the dominant categories and the procedures through which errors emerge, the study seeks to provide evidence for more targeted instruction in factorization, sign handling, symbolic interpretation, equivalence, and algebraic reasoning within comparable under-supported school contexts.

## METHOD

### Research Design

This study employed a descriptive error-analysis design combining qualitative examination of students' written responses with quantitative frequency analysis. The qualitative component focused on identifying and interpreting the mathematical errors evident in students' solution procedures when simplifying rational algebraic expressions. Students' written work was examined to determine the conceptual, symbolic, reasoning, procedural, and technical difficulties underlying their responses. The quantitative component was used to summarize the occurrence of the identified errors through frequencies and percentages. This combined

approach was considered appropriate because the purpose of the study was not only to determine whether students produced correct or incorrect answers, but also to characterize the types of errors occurring in their solution processes and describe how frequently these errors appeared across the dataset. The interpretation of errors was guided by the classification framework adapted from Aguilera et al. (2020), with attention to conceptual, procedural, symbolic, and technical aspects of algebraic performance.

### **Participants and Research Setting**

The participants were 28 students enrolled in an Algebra I class at a rural secondary school in the Democratic Republic of the Congo during the 2025 academic year. To be included in the study, students were required to be enrolled in the selected Algebra I class, to have received prior instruction related to algebraic expressions and factorization, and to complete the research test during the data-collection period.

The study was conducted in a rural secondary school located in [province/territory, if appropriate to disclose], Democratic Republic of the Congo.. The site was selected because [state the actual reason, e.g., accessibility to the researcher, relevance to the research problem, evidence of students' difficulties with algebra, or limited previous research in this context]. The selection of this setting was particularly relevant because research describing students' error patterns in rational algebraic-expression simplification remains limited in rural Congolese secondary-school contexts.

### **Instrument**

Data were collected using an evaluative test consisting of 10 items on the simplification of rational algebraic expressions. The items were designed to elicit students' written solution procedures rather than only their final answers, thereby allowing errors to be identified at different stages of algebraic manipulation. The tasks assessed mathematical knowledge and procedures required for rational-expression simplification, including factorization, manipulation of algebraic signs and symbols, application of algebraic properties, operations involving rational expressions, recognition of algebraic structure, and the use of appropriate simplification procedures. Each of the 28 students responded to the 10 items, yielding a maximum dataset of 280 item responses.

### **Data Collection Procedure**

The test was administered to the 28 participating students in the 2025 academic year. Students were instructed to show their solution procedures so that the reasoning underlying each response could be examined. The completed scripts were collected and subsequently organized by participant and item for analysis. Across the 28 students and 10 test items, a total of 280 responses were available for examination.

### **Data Analysis**

The unit of analysis was each student response to each test item, including both the final answer and the written mathematical procedure used to obtain it. Thus, 28 students responding to 10 items generated 280 units of response analysis. Each response was first classified as correct, unanswered, or containing at least one identifiable error. The written procedures in erroneous responses were then examined to determine the nature of the error.

An error was identified when a student's written procedure contained a mathematical transformation, interpretation, operation, or application of a rule that was inconsistent with accepted algebraic principles and resulted in, or contributed to, an invalid step or conclusion. Errors were subsequently coded according to the adapted classification framework used in this study, which included errors associated with language or symbolic interpretation, deficient prior knowledge, incorrect associations or rigidity in thinking, application of irrelevant rules or strategies, misuse of data, distorted theorems or definitions, and technical execution (Boyce & Aguilera, 2020). Because different mistakes could occur within a single solution, one response could receive more than one error code when distinct errors were present. This coding approach is consistent with the dataset: although 136 responses contained at least one error, a total of 288 error instances were identified across those responses.

After qualitative coding, frequencies were calculated for each error category and converted into percentages to describe their distribution across the dataset. Overall response outcomes were also summarized: of the 280 responses, 136 contained at least one error, 106 were correct, and 38 were unanswered. Frequencies and percentages were used descriptively to identify the errors that appeared most often and to compare the relative prominence of error categories.

## RESULTS

A total of 28 students completed the 10-item test, producing 280 responses for analysis. Of these responses, 106 (37.9%) were answered correctly, 38 (13.6%) were unanswered, and 136 (48.6%) contained at least one error. Because a student response could contain more than one type of error, the number of coded error occurrences was greater than the number of responses containing errors. Across the written solutions, 228 error occurrences were identified and classified according to the error categories used in this study.

The analysis covered all 10 test items. However, the following subsections present selected student responses that most clearly illustrate the recurrent difficulties identified during the analysis. These examples include errors in factorization, sign manipulation, algebraic reasoning, and the application of simplification procedures.

*Exercise (a).* Simplify the following expression.

$$\frac{(x^3 + 8)}{-x^2 + 4}$$

In Item (a), two recurrent difficulties were identified from the students' written solutions. The first concerned the incorrect handling of the negative sign in the denominator. This error was observed in 10 of the 28 students (35.7%). The second involved incorrect factorization of the numerator and was observed in 11 students (39.3%). These errors indicate that some students were able to recognize the need for factorization but did not correctly preserve the algebraic structure and signs during the transformation.

$$\text{a) } \frac{x^3 + 8}{-x^2 + 4} = \frac{(x+2)(x^2 + 2x + 4)}{(x-2)(x+2)} = \frac{(x^2 + 2x + 4)}{(x-2)} = \frac{(x+2)^2}{(x-2)}$$

**Figure 1.** Student response showing errors in sign handling and factorization in Item (a)

The response illustrated in Figure 1 shows an attempt to factor both the numerator and denominator. However, the student incorrectly handled the sign in the denominator and also produced an incorrect factorization of the numerator. As a result, the subsequent cancellation led to a non-equivalent expression.

*Exercise (d).* Simplify the following expression.

$$\frac{1 - b^2}{(1 + bx)^2 - (b + x)^2}$$

The written solutions to Item (d) also revealed difficulties in factorization and the cancellation of algebraic factors. Some students recognized components of the required factorization but applied the transformations inconsistently. In particular, factors in the numerator and denominator were manipulated or cancelled before the expressions had been correctly rewritten into equivalent factored forms.

$$\text{d) } \frac{p(p+q)(p^3 - q^3)}{(p-q)(p^2 - q^2)} = \frac{p^2 + pq + (p-q)(p^2 - pq + q^2)}{(p-q)(p^2 - pq + q^2)} = p^2 + pq$$

**Figure 2.** Example of incorrect factorization and cancellation of algebraic factors in Item (d)

Figure 2 shows that the student attempted to factor the expressions before simplifying them, but the resulting factors were not consistently maintained throughout the procedure.

$$\begin{aligned}
 d) \quad \frac{p(p+q)(p^3-q^3)}{(p-q)(p^2-q^2)} &= \frac{p(p+q)(p^3-q^3)}{(p-q)(p+q)(p-q)} = \frac{(p^4-q^4)}{(p-q)(p-q)} \\
 &= \frac{(p^2+q^2)(p^2-q^2)}{(p-q)(p-q)} = \frac{(p^2+q^2)((p+q)(p-q))}{(p-q)(p-q)} \\
 &= \frac{(p^2+q^2)(p+q)}{(p-q)} = \frac{(p+q)^3}{p-q}
 \end{aligned}$$

**Figure 3.** Example of an incorrect simplification procedure involving factor cancellation in Item (d)

A similar difficulty is visible in Figure 3. Although the student identified several potentially useful factors, the cancellation procedure was applied incorrectly, resulting in an expression that was not equivalent to the original one.

#### Exercise (h)

A particularly frequent difficulty was observed in Item (h). Fourteen of the 28 students (50.0%) demonstrated errors when simplifying the notable products appearing in the denominator. The written responses indicate difficulties in expanding or factoring the expressions correctly before proceeding with simplification.

$$d) \quad \frac{1-b^2}{(1+bx)^2-(b+x)^2} = \frac{(1+b)(1-b)}{[(1+bx)+(b+x)-(b+x)]}$$

**Figure 4.** Example of an incorrect application of a notable-product identity in Item (h)

As shown in Figure 4, the student attempted to rewrite the numerator and denominator using algebraic identities, but the denominator was not transformed correctly.

$$\begin{aligned}
 h) \quad \frac{1-b^2}{(1+bx)^2-(b+x)^2} &= \frac{1-b^2}{(1+2bx+b^2x^2)-b^2+2bx+x^2} \\
 &= \frac{1-b^2}{1+b^2x^2-b^2+x^2}
 \end{aligned}$$

\* Le signe (+) est (-)

**Figure 5.** Example of an incorrect expansion of the denominator in Item (h)

Figure 5 provides another example of difficulty in dealing with the squared expressions in the denominator. The student expanded the terms but did not maintain the correct signs and structure of the expression. These responses show that notable products and their use in rational-expression simplification represented an important source of difficulty for the participants.

#### Analysis Categories

The identified errors were further examined according to three broad aspects of students' algebraic work: conceptual errors, mathematical reasoning errors, and procedural errors. These categories were used to describe the mathematical characteristics of the errors observed in the students' written solutions.

*Conceptual Error Analysis*

Conceptual errors were identified when students incorrectly applied algebraic properties, identities, or definitions required for simplifying rational expressions. These errors were particularly visible in factorization and in the manipulation of algebraic expressions involving signs and polynomial identities.

$$\begin{aligned}
 \text{a) } \frac{x^3 + 8}{-x^2 + 4} &= \frac{(x+2)(x^2 - 2x + 4)}{-(x^2 - 4)} \\
 &= \frac{(x+2)(x^2 - 2x + 4)}{-(x-2)(x+2)} = -\frac{(x^2 - 2x + 4)}{x-2} \\
 &= \frac{-(x-2)(x-2)}{x-2} = \frac{-(x-2)^2}{x-2} = -(x-2)
 \end{aligned}$$

**Figure 6.** Example of an incorrect application of the sum-of-cubes factorization identity

Figure 6 illustrates a factorization error. The student recognized that the expression could be factored but incorrectly formed one of the factors. Consequently, the following simplification steps were based on an incorrect algebraic identity and produced an invalid result.

Handwritten work for Figure 7:
 
$$\begin{aligned}
 \text{e) } 1 + \frac{1}{2 + \frac{1}{y}} &= 1 + \frac{1}{\frac{2y+1}{y}} \\
 &= 1 + \frac{1}{2y^2 + y} \\
 &= \frac{2y^2 + y + 1}{2y^2 + y}
 \end{aligned}$$
 The final result is circled in the image.

**Figure 7.** Example of an incorrect manipulation of terms in a rational algebraic expression

In Figure 7, the student attempted to simplify a rational expression through a sequence of transformations but did not preserve the correct operational structure. The terms in the denominator were combined incorrectly, producing a non-equivalent expression.

Handwritten work for Figure 8:
 
$$\begin{aligned}
 \text{j) } \frac{m^2 - a}{am - m^3} &= \frac{(m-a)(m+a)}{-m(m^2 - a)} \\
 &= \frac{(m-a)(m+a)}{-m(m-a)(m+a)} = -\frac{1}{m}
 \end{aligned}$$
 The final result is  $-\frac{1}{m}$ .

**Figure 8.** Example of incorrect sign handling during factorization and simplification

Figure 8 shows another conceptual difficulty associated with factorization and signs. Although the student identified a possible factored form, the relationship between the factors and the negative sign was not handled consistently. This changed the value of the expression during simplification.

*Mathematical Reasoning*

Errors in mathematical reasoning were observed when students selected inappropriate relationships among terms or failed to recognize the structural relationships necessary for simplification. These responses differed from simple computational mistakes because the difficulty was evident in the way the expression was reorganized and interpreted.

Handwritten work for Figure 9:
 
$$\begin{aligned}
 \text{i) } \frac{x^2 - 6x + 9}{x^3 - 9x^2 + 27x - 27} &\rightarrow \frac{(x-3)(x-3)}{(x^3 + 27x) - (9x^2 + 27)} \rightarrow \frac{(x-3)(x-3)}{x(x^2 + 27) - (9x^2 + 27)}
 \end{aligned}$$
 The final expression is circled in the image.

**Figure 9.** Example of an inappropriate factorization strategy in simplifying a rational algebraic expression



The response in Figure 9 shows that the student attempted to identify common algebraic structures but did not establish a valid relationship between the factors in the numerator and denominator. The resulting transformation therefore did not support a correct simplification.

$$\begin{aligned} \text{d) } \frac{p(p+q)(p^3-q^3)}{(p-q)(p^2-q^2)} &= \frac{p(p+q)(\cancel{p^2-q^2})(p+q)}{(p-q)(p+q)(\cancel{p-q})} \\ &= p(p^2-q^2) \\ &= \frac{p(p+q)(p-q)}{(p-q)} \end{aligned}$$

**Figure 10.** Example of an incorrect association and cancellation of factors in the denominator

Figure 10 illustrates an inappropriate association of terms in the denominator. The student attempted to reorganize and cancel factors, but the factors being treated as equivalent were not algebraically identical. This resulted in an incorrect final expression.

Another reasoning difficulty observed in the written solutions involved powers of algebraic expressions. Some students treated a sum of squared terms as if it were equivalent to the square of a sum. Such transformations altered the algebraic structure of the expression and prevented a valid simplification.

#### *Analysis of Procedural Errors*

Procedural errors occurred when students selected or executed an inappropriate algebraic rule or algorithm. These errors included the application of irrelevant simplification strategies, incorrect manipulation of expressions, and technical mistakes in handling parentheses and negative signs.

$$\begin{aligned} \text{g) } \frac{-x^2+1}{x^3+3x^2-x-3} &= \frac{-x^2+1}{(x^3+3x^2x)-3} \\ &= \frac{-x^2+1}{x(x^2+3x-1)-3} \\ &= \frac{-x^2+1}{x[(x^2+3x-4)]} \\ &= \frac{-x^2+1}{x[(x+4)(x-1)]} = \frac{-x^2+1}{x[(x+4)(x-1)]} \end{aligned}$$

**Figure 11.** Example of an irrelevant rule or strategy applied during algebraic simplification

Figure 11 shows a student applying a procedure that was not appropriate for the structure of the given rational expression. Although several algebraic transformations were attempted, the procedure did not preserve equivalence and therefore led to an incorrect solution.

$$\begin{aligned} \text{h) } \frac{1-b^2}{(1+bx)^2-(b+x)^2} &= \frac{1-b^2}{(1+2bx+b^2x^2)-b^2+2bx+x^2} \\ &= \frac{1-b^2}{1+b^2x^2-b^2+x^2} \\ &= \frac{1-b^2}{1+b^2x^2-b^2+x^2} \end{aligned}$$

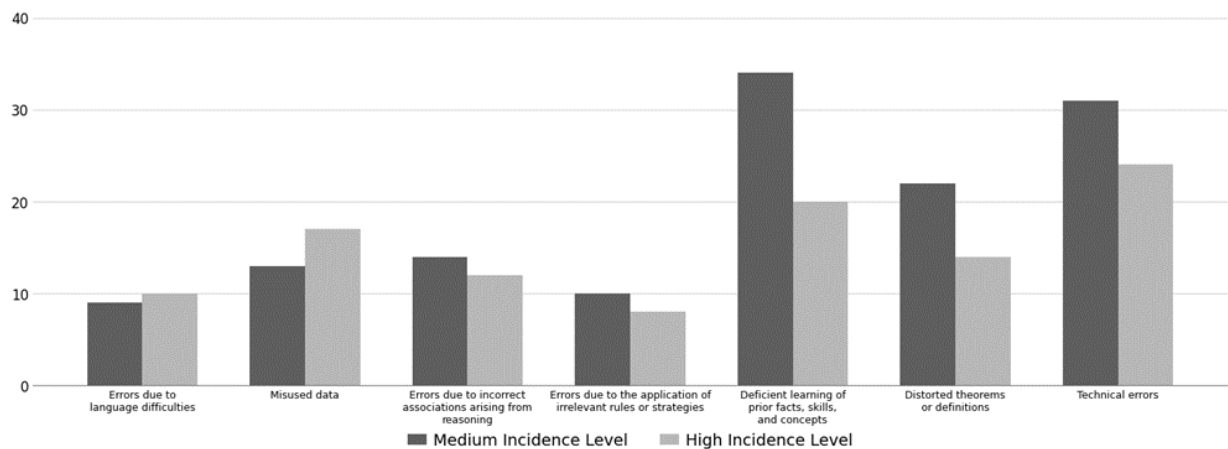
**Figure 12.** Example of a technical error in distributing a negative sign across parentheses

In Figure 12, the student expanded the denominator but did not correctly account for the negative sign preceding the parenthetical expression. The signs of the affected terms should have changed when the parentheses were removed. This error illustrates a difficulty in executing an otherwise relevant algebraic procedure.

#### *Distribution of Error Categories*

The frequency analysis showed differences in the occurrence of the seven error categories. Across the students' written solutions, 228 coded error occurrences were identified. Technical errors were the most frequent category, with 55 occurrences (24.1%), followed closely by errors associated with deficient prior knowledge of facts, skills, and concepts, with 54 occurrences (23.7%). Misuse of data accounted for 30 occurrences (13.2%), while incorrect association or rigidity in thinking and distorted theorems or definitions

each accounted for 26 occurrences (11.4%). Errors related to language difficulties occurred 19 times (8.3%), whereas the application of irrelevant rules or strategies occurred 18 times (7.9%).



**Figure 13.** Comparison of medium- and high-incidence error counts across error categories

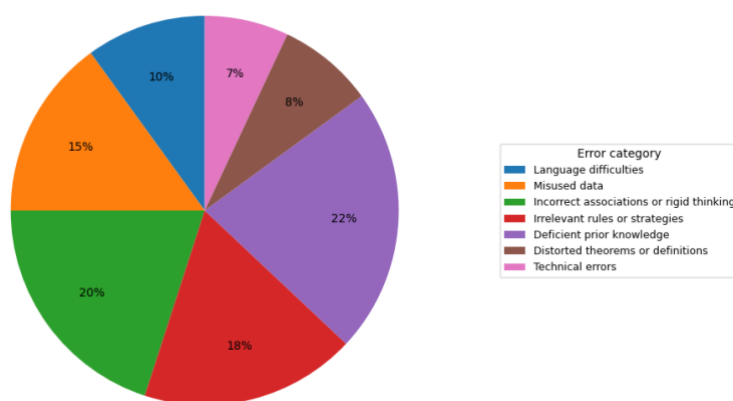
The graphical distribution in Figure 13 shows that technical errors and deficiencies in prior knowledge were particularly prominent in students' written work. These categories included difficulties in factorization, algebraic manipulation, and the execution of previously learned procedures.

**Table 1.** Distribution of Error Categories and Students' Response Outcomes

| Error Categories                                                     | Medium     | High       | Total      | Percentage    |
|----------------------------------------------------------------------|------------|------------|------------|---------------|
| Error due to language difficulties                                   | 9          | 10         | 19         | 8.3           |
| Error due to deficient learning of prior facts, skills, and concepts | 34         | 21         | 54         | 23.7          |
| Error due to incorrect association or rigidity in thinking           | 14         | 12         | 26         | 11.4          |
| Error due to the application of irrelevant rules or strategies       | 10         | 8          | 18         | 7.9           |
| Misused data                                                         | 13         | 17         | 30         | 13.2          |
| Disorted theorems or definitions                                     | 12         | 14         | 26         | 11.4          |
| Technical errors                                                     | 31         | 24         | 55         | 24.1          |
| <b>Number of coded error occurrences</b>                             | <b>123</b> | <b>105</b> | <b>228</b> | <b>100.0</b>  |
| Items solved correctly                                               |            |            | 106        | 37.9*         |
| Items not solved                                                     |            |            | 38         | 13.6*         |
| Items with at least one error                                        |            |            | 136        | 48.6*         |
| <b>Total number of responses</b>                                     |            |            | <b>280</b> | <b>100.0*</b> |

*Note:* Percentages for the seven error categories were calculated using 228 coded error occurrences as the denominator. Percentages marked with an asterisk were calculated using the 280 item responses as the denominator.





**Figure 14.** Percentage distribution of error categories in simplifying rational algebraic expressions

The existing pie chart should be updated to correspond to the corrected percentages reported in Table 1: technical errors (24.1%), deficient prior knowledge (23.7%), misuse of data (13.2%), incorrect association or rigidity in thinking (11.4%), distorted theorems or definitions (11.4%), language difficulties (8.3%), and application of irrelevant rules or strategies (7.9%).

Overall, the results indicate that students' difficulties were concentrated particularly in technical execution and prerequisite algebraic knowledge. The representative written responses further show that these difficulties appeared in factorization, handling of signs, recognition of algebraic structure, and the appropriate selection and execution of simplification procedures.

## DISCUSSION

The findings show that difficulties in simplifying rational algebraic expressions were widespread among the participating students. Of the 280 responses analyzed, 136 (48.6%) contained at least one error, indicating that almost half of the students' item-level responses involved an incorrect algebraic transformation or procedure. Moreover, 228 individual error occurrences were identified across these responses, showing that a single incorrect response could involve more than one difficulty. This pattern supports the view that errors in algebra are often systematic rather than isolated computational slips, because they may reflect incomplete conceptual knowledge or procedures that students repeatedly apply inappropriately (Hoth et al., 2022). Similar patterns have been reported in studies of algebraic simplification, where students' errors are associated with difficulties in interpreting symbols, maintaining algebraic structure, and selecting appropriate transformation procedures (Lim, 2010; Moru & Mathunya, 2022).

The most frequent categories in the present study were technical errors and errors associated with deficient prior knowledge of mathematical facts, skills, and concepts. These findings suggest that students' difficulties were strongly related to prerequisite knowledge needed for algebraic manipulation. Simplifying rational expressions requires students to coordinate knowledge of fractions, factorization, signs, algebraic identities, and equivalence. Weaknesses in any of these areas may therefore interfere with the entire simplification process. Chrysostomou and Christou (2019) similarly reported that misconceptions involving fractions, equivalent fractions, and fraction simplification may persist and subsequently hinder learning involving rational algebraic expressions. Moru and Mathunya (2022) further showed that students' simplification errors frequently arise from overgeneralization of previously learned rules and misunderstanding of algebraic meaning. Thus, the dominance of prior-knowledge and technical errors in this study indicates that many students had not yet developed sufficiently stable foundations for manipulating more complex rational expressions.

Factorization was one of the most visible sources of difficulty in the students' written work. In Item (a), for example, incorrect factorization of the numerator occurred in 11 students, while Item (h) revealed substantial difficulty with notable products and denominator simplification. These findings are consistent with the argument that algebraic simplification requires students to recognize the structural organization of an expression before carrying out transformations. Students who perceive an expression mainly as a sequence of symbols may apply familiar procedures without recognizing whether the required factorization is mathematically valid (Ardiansari et al., 2020; Qetrani et al., 2021). This distinction between procedural manipulation and structural understanding is particularly relevant to rational expressions because valid

cancellation is possible only after the numerator and denominator have been represented appropriately in factored form. The errors observed in the present study therefore indicate not only weaknesses in carrying out factorization procedures but also difficulties in recognizing when and how particular algebraic structures can be transformed.

Sign manipulation constituted another important difficulty. In Item (a), 10 students incorrectly handled the sign in the denominator, while other examples showed problems distributing negative signs across parentheses. Previous studies have similarly identified negative signs, brackets, and subtraction structures as persistent sources of algebraic error (Ardiansari et al., 2020; Barana, 2021; Ottmar et al., 2023). Such errors may arise because the minus sign can carry different meanings depending on its position in an expression, while bracketed expressions require students to coordinate signs with the structure of the entire expression. When these meanings are not sufficiently understood, students may perform transformations that appear procedurally familiar but do not preserve equivalence. This observation is especially important in rational-expression simplification, where a single sign error in a factor or denominator can alter all subsequent steps and produce an incorrect final expression.

The mathematical-reasoning and procedural errors identified in the students' work provide further evidence that difficulty was not limited to basic calculation. Several students formed inappropriate associations between terms, applied irrelevant rules, or cancelled expressions that were not valid common factors. These errors are consistent with the classification proposed by Aguilera et al. (2022), in which mathematical difficulties may reflect conceptual, procedural, and cognitive dimensions. They also support findings that students sometimes transfer previously learned schemas into new algebraic contexts without considering whether those procedures remain appropriate (Moru & Mathunya, 2022). For example, overgeneralizing arithmetic rules or equation-solving procedures can lead learners to manipulate algebraic expressions on the basis of surface similarities rather than structural relationships. Barana (2021) likewise reported oversimplification and inaccurate operations with algebraic expressions, whereas Qetrani et al. (2021) emphasized the importance of structure sense in recognizing equivalent forms. Taken together, these studies help explain why some students in the present investigation could initiate an apparently relevant procedure but were unable to maintain mathematical equivalence throughout the solution.

The results also highlight the importance of equivalence as an underlying concept in rational-expression simplification. Simplification is not simply a process of producing a shorter expression; every transformation must preserve the mathematical value and structure of the original expression. Students who have an unstable understanding of equivalence may therefore regard simplification as a sequence of permitted symbolic operations rather than as the construction of an equivalent expression. Previous research has shown that students frequently interpret equality and equivalence operationally rather than relationally, which may hinder their ability to recognize whether two algebraic expressions represent the same mathematical object (Ardiansari et al., 2020; Chan et al., 2022). The non-equivalent transformations observed in the present study, particularly during factorization and cancellation, suggest that strengthening equivalence reasoning should accompany instruction on procedural techniques.

From an instructional perspective, these findings indicate that teaching rational algebraic expressions should not focus exclusively on memorizing factorization formulas or simplification algorithms. Students need opportunities to explain why a transformation is valid, compare alternative solution strategies, and identify the algebraic structure that justifies cancellation or factorization. Chan et al. (2022) noted that instruction dominated by rule demonstration may provide limited opportunities for students to articulate and evaluate their reasoning. Conversely, learning environments that emphasize discussion, representation, and targeted scaffolding may help teachers detect misconceptions before they become persistent procedures (Eroğlu, 2023; Mouhayar, 2021). In the context of the present study, particular instructional attention should be directed toward factorization, notable products, manipulation of negative signs, interpretation of algebraic symbols, and verification of equivalent expressions.

The rural Congolese setting also gives the findings contextual significance. However, the observed errors should not be attributed directly to rurality because this study did not measure the effects of school resources, teacher availability, instructional time, or other contextual variables. Research nevertheless suggests that students' opportunities to learn and the type of instructional support available to them can influence the persistence of foundational mathematical difficulties (Kinach, 2024; Mouhayar, 2021; Rahmawati and Neviyarni, 2026). The present findings therefore provide an initial description of algebraic difficulties within a context that remains underrepresented in mathematics-education research. Rather than establishing rural schooling as a cause of error, the study identifies specific areas of student difficulty that can inform subsequent investigations of how instructional and contextual factors shape algebra learning in rural Congolese schools.

Overall, the study extends previous error-analysis research by documenting the forms of conceptual, reasoning, and procedural difficulty evident in rural Congolese students' simplification of rational algebraic

expressions. The convergence between the present findings and previous studies indicates that factorization, prior knowledge, sign handling, symbolic structure, and equivalence remain central challenges in algebra learning (Ardiansari et al., 2020; Lim, 2010; Moru & Mathunya, 2022; Husna, et al, 2024; Baggas, et al. 2026). At the same time, examining these difficulties in the Democratic Republic of the Congo contributes context-specific evidence from a setting that has received comparatively limited attention in the literature. These findings can therefore serve as a basis for developing more focused instructional interventions and for future studies examining how students' conceptual understanding and procedural fluency in rational algebraic expressions develop across different Congolese educational settings.

## CONCLUSION

This study identified recurring errors in rural Congolese students' simplification of rational algebraic expressions. Of the 280 item responses analyzed, 136 (48.6%) contained at least one error. The analysis showed that students' difficulties were particularly associated with technical execution and insufficient prior knowledge of mathematical facts, skills, and concepts. Representative written responses further revealed recurring problems in factorization, manipulation of negative signs, application of algebraic identities, recognition of algebraic structure, and the selection of appropriate simplification procedures. These findings indicate that students' errors were not limited to isolated computational mistakes but also reflected weaknesses in conceptual understanding, mathematical reasoning, and procedural fluency.

The study contributes to mathematics education research by providing empirical evidence on algebraic error patterns in a rural secondary-school context in the Democratic Republic of the Congo, a setting that remains relatively underrepresented in research on algebra learning. Pedagogically, the findings suggest that instruction on rational algebraic expressions should place greater emphasis on prerequisite knowledge, particularly factorization and algebraic identities, while also helping students understand why each transformation preserves equivalence. Classroom activities that require students to explain solution steps, compare alternative procedures, identify incorrect transformations, and verify equivalent expressions may help reduce the persistence of these errors. The findings may also provide teachers with a practical basis for diagnosing students' difficulties and designing targeted remedial instruction.

Future studies should involve larger and more diverse samples from rural and urban schools in different regions of the Democratic Republic of the Congo to determine whether similar error patterns occur across educational settings. Further research should also examine the relationship between students' errors and factors such as prior mathematical achievement, instructional practices, language of instruction, and opportunities to learn. Interviews or think-aloud procedures could complement written-response analysis by providing deeper evidence about the reasoning underlying particular errors. In addition, intervention studies are needed to evaluate whether instructional approaches emphasizing structural understanding, equivalence, factorization, and explicit error analysis can improve students' accuracy and conceptual understanding in simplifying rational algebraic expressions.

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